Derivation of the Effective Pion-Nucleon Lagrangian within Heavy Baryon Chiral Perturbation Theory

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Abstract

We develop a method for constructing the Heavy Baryon Chiral Perturbation Theory (HBChPT) Lagrangian, to a given chiral order, within HBChPT. We work within SU(2) theory, with only the pion field interacting with the nucleon. The main difficulties, which are solved, are to develop techniques for implementing charge conjugation invariance, and for taking the nucleon on shell, both within the non-relativistic formalism. We obtain complete lists of independent terms in $\mathcal{L}_{\text{HBChPT}}$ through $\text{O}(q^3)$ for off- shell nucleons. Then, eliminating equation-of-motion (eom) terms at the relativistic and nonrelativistic level (both within HBChPT), we obtain $\mathcal{L}_{\text{HBCHPT}}$ for on-shell nucleons, through $\text{O}(q^3)$. The extension of the method (to obtain on-shell $\mathcal{L}_{\text{HBChPT}}$ within HBChPT) to higher orders is also discussed.

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1 Introduction

The theory of strong interactions, Quantum Chromodynamics (QCD), has no known exact solutions at low energies (referred to as the nonperturbative regime). Among the non-perturbative methods used, as an alternative, effective field theories (EFT) have been quite popular and successful. The EFT that has now become quite popular is generically referred to as Chiral Perturbation Theory (ChPT). ChPT was studied systematically first by Gasser and Leutwyler [1] for purely mesonic systems. It was later extended to include baryons (nucleons) by Gasser, Sainio and Svarc [2] (referred to as Baryon ChPT [BChPT]). The nonrelativistic limit of BChPT was taken by Manohar and Jenkins, and the theory is referred to as Heavy BChPT [HBChPT] [3].

In the method given in the recent literature, the nonrelativistic Lagrangian of Heavy Baryon Chiral Perturbation Theory ($\mathcal{L}_{\mathrm{HBChPT}}$) is obtained by a $\frac{1}{\mathrm{m}}$-reduction [4] of the relativistic Lagrangian $\mathcal{L}_{\mathrm{BChPT}}$. This first requires a complete representation of $\mathcal{L}_{\mathrm{BChPT}}$. It would be more efficient if one could construct $\mathcal{L}_{\mathrm{HBChPT}}$ directly, by working entirely within the framework of the nonrelativistic (effective) field theory, without going through the $\frac{1}{\mathrm{m}}$-reduction (or Foldy-Wouthysen's transformation [5]).

The main goal in this paper is to develop a method to construct $\mathcal{L}_{\mathrm{HBChPT}}$ directly within the framework of HBChPT for processes involving a single off-shell nucleon and arbitrary number of pions. [For this paper, weak and electromagnetic interactions will not be included in the EFT.] Such an off-shell Lagrangian can find applications in nuclear HBChPT as applied to e.g., pion double charge exchange scattering off a nuclear target (that involves nucleons in a bound state). Much of the process of implementing the requirements of chiral symmetry, hermiticity, and Lorentz symmetries proceeds by methods similar to those already in the literature. However, a new technique is required and developed, for implementing charge conjugation invariance within the nonrelativistic framework.

In this paper, the method developed will be used for generating complete lists of independent terms in the Lagrangian, of $\mathrm{O}(q^2)$ and $\mathrm{O}(q^3)$. The extension of the method to construction of $\mathrm{O}(q^4)$ terms will be reported elsewhere. We first construct the lists of independent off-shell terms; then we use equation of motion techniques to generate the list of independent on-shell terms. This requires a new method of embedding the effects of relativistic equations

of motion in the HBChPT formalism.

In [6], EM have obtained HBChPT on-shell lists through $O(q^3)$ by $\frac{1}{m}$-reduction of $\mathcal{L}_{BChPT}$. Our method produces the same on-shell lists, as well as the full off-shell lists, but working entirely within HBChPT. Our results are shown to be identical to those of EM for the on-shell lists. We also demonstrate how to extend the (on-shell) method to higher orders.

The basic meson and nucleon operators used for the construction of $\mathcal{L}_{(H)BChPT}$, referred to as "building blocks", are introduced in Section **2**, in which we also implement chiral and Lorentz symmetries, and hermiticity. Our method for including charge conjugation invariance is given in Section **3**. The following section reduces the number of independent terms, using several algebraic identities. The complete lists of off-shell terms of the type $A^{(n)}$ (See (4)), for chiral order $n = 2, 3$ are given in Section **5**. In Section **6**, a method is given for obtaining the on-shell $\mathcal{L}_{HBChPT}$ within HBChPT; using it, the on-shell limit of the HBChPT terms up to $O(q^3)$ is taken. Comparison is made with EM's paper [6] in which they obtain the same on-shell terms, but by following the longer $\frac{1}{m}$-reduction formalism. Some concluding remarks are given in Section **7**.

2   Preliminaries

The nonrelativistic Lagrangian ($\mathcal{L}_{HBChPT}$) is constructed in terms of building blocks which are of two kinds. The first kind are field operators constructed out of pion fields and derivatives, referred to as pion-field-dependent building blocks. There are four of them: vector, axial-vector, scalar and pseudo-scalar. [Since we do not consider vector or axial-vector mesons (or resonances) and photons in this paper, there is no need to include a tensor field operator. For the latter, see [7].] The second kind are the baryon building blocks, which act only on the baryon fields. There are three of them which survive the nonrelativistic reduction of the five types of Dirac tensors: a vector and an axial-vector, and the unit scalar **1**.

We use the non-linear realization of chiral symmetry in which the nucleon isospinor transforms non-linearly (with respect to pions). To implement this, one introduces a field operator $u \equiv \sqrt{U}$, where $U = e^{\frac{i\phi}{F_\pi}}$. Here, $\phi \equiv \vec{\pi} \cdot \vec{\tau}$, where $\vec{\pi}$ is the pion field, $\vec{\tau}$ is the nucleon isospin operator, and F_π is pion decay constant.

The following are the pion-field dependent building blocks that will be used for construction of $\mathcal{L}_{\text{HBChPT}}$, listed by their Lorentz property. It is instructive to note that because these four building blocks are independent of the Dirac tensors, they are the same in BChPT and HBChPT.

$$\text{vector}: \ \mathrm{D}_\mu = \partial_\mu + \Gamma_\mu, \ \text{where } \Gamma_\mu \equiv \frac{1}{2}[u^\dagger, \partial_\mu u];$$

$$\text{axial} - \text{vector}: \ u_\mu \equiv i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger);$$

$$\text{scalar}: \ \chi_+ \equiv u^\dagger \chi u^\dagger + u \chi^\dagger u,$$

$$\text{pseudo} - \text{scalar}: \ \chi_- \equiv u^\dagger \chi u^\dagger - u \chi^\dagger u,$$

$$(1)$$

where $\chi \equiv \mathrm{M}_\pi^2$ for this paper. [1] As regards the chiral orders of the four pion-field-dependent building blocks, using the facts that the field operators $U, u \equiv \mathcal{O}(1)$ and that $\partial_\mu \equiv \mathcal{O}(q)$, one sees:

$$u_\mu, \mathrm{D}_\mu \equiv \mathcal{O}(\mathrm{q}), \ \chi_\pm \equiv \mathcal{O}(\mathrm{q}^2). \qquad (2)$$

All four of the pion-field dependent building blocks are constructed in such a way that they each transform homogeneously under the non-linear chiral transformation K (defined via $\mathrm{R}u\mathrm{K}^\dagger = \mathrm{K}u\mathrm{L}^\dagger$, where R, L are SU(2) matrices corresponding to independent chiral rotations of the right (R)- and left (L)- handed components of the quark field). Invariance under the non-linear realization of chiral symmetry imposes the constraint that $\mathcal{L}_{\text{(H)BChPT}}$ can not contain D_μ in the combination $\langle [\mathrm{D}_\mu, \ ]_+ \rangle$, where $\langle \ \rangle$ denotes the trace on nucleon isospin. With this restriction, the Goldstone bosons are guaranteed to remain massless in the chiral limit, and the Ward identities are automatically satisfied by the Lorentz-invariant and anomaly-free generating functional [8].

A bilinear $\bar{\psi}\mathcal{O}\psi$ in HBChPT is rewritten in terms of:

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[1] In general, $\chi \equiv 2B(s + ip)$, where s and p are the external scalar and pseudoscalar fields respectively. The constant B is related to the quark condensate (See [1]).

$$\mathrm{H} \equiv e^{imv \cdot x} \frac{1}{2}(1 + \slashed{v})\psi,$$

$$\mathrm{h} \equiv e^{imv \cdot x} \frac{1}{2}(1 - \slashed{v})\psi, \tag{3}$$

referred to as the "upper" and "lower" components of ψ respectively. (Here the parameter v_μ represents the nucleon velocity, and m is the nucleon mass.) The bilinear then becomes:

$$\bar{\psi}\mathcal{O}\psi = \bar{\mathrm{H}}A\mathrm{H} + \bar{\mathrm{h}}B\mathrm{H} + \bar{\mathrm{H}}\gamma^0 B^\dagger \gamma^0 \mathrm{h} - \bar{\mathrm{h}}C\mathrm{h}. \tag{4}$$

A systematic path integral derivation for $\mathcal{L}_{\mathrm{HBChPT}}$ based on a paper by Mannel et al [9], starting from $\mathcal{L}_{\mathrm{BChPT}}$ was first given by Bernard et al [4]. As shown by them, after integrating out h from the generating functional, one arrives at $\mathcal{L}_{\mathrm{HBChPT}}$:

$$\mathcal{L}_{\mathrm{HBChPT}} = \bar{\mathrm{H}}\Big(A + \gamma^0 B^\dagger \gamma^0 C^{-1} B\Big)\mathrm{H}, \tag{5}$$

an expression in the upper components only i.e. for non-relativistic nucleons.

The baryon building blocks act on the baryon fields; for terms in $\mathcal{L}_{\mathrm{HBChPT}}$ of the form $\bar{\mathrm{H}}(...)\mathrm{H}$ they are defined in terms of the nucleon velocity v_μ and the Pauli-Lubanski spin operator $\mathrm{S}_\mu \equiv \frac{i}{2}\gamma^5\sigma^{\mu\nu}v_\nu$. For such terms all five types of Dirac tensors can be reduced to v_μ, S_ν (and $\mathbf{1}$).

Terms of the $\mathcal{L}_{\mathrm{(H)BChPT}}$ constructed from products of building blocks will automatically be chiral invariant. Symbolically, a term in $\mathcal{L}_{\mathrm{HBChPT}}$ can be written as just a product of the building blocks [(1) and v_μ, S_ν] to various powers (omitting $\mathrm{H}, \bar{\mathrm{H}}$ as will be done in the rest of the paper):

$$\mathrm{D}_\alpha^m u_\beta^n \chi_+^p \chi_-^q v_\sigma^l \mathrm{S}_\kappa^\mathrm{r} \equiv (\mathrm{m, n, p, q}) \equiv \mathrm{O}(\mathrm{q^{m+n+2p+2q}}). \tag{6}$$

The chiral order follows from (2); note that only pion-field dependent building blocks contribute to the chiral order (if constructing B-type terms, then one also uses $\gamma^5 \equiv \mathrm{O}(q)$). The isospin trace is not written because only the number of the building blocks is important.

In (6), using the algebra of S's, one can show that it is sufficient to consider $r = 0$ or 1 only (See [8]).

Now, we have considered $\mathrm{O}(\phi^{2n})$ (even intrinsic parity) and $\mathrm{O}(\phi^{2n+1})$ (odd intrinsic parity) terms separately as it helps in ensuring completeness. The

following is used: $u_\mu, \chi_- \equiv O(\phi^{2n+1})$ and $D_\mu, \chi_+ \equiv O(\phi^{2n}), n = 0, 1, 2, ...$ ($\phi \equiv$ mesons). Hence, if one were to construct $O(q^3)$ counter terms then the allowed 4-tuples (m,n,p,q) are given in Table 1.

We discuss how to exploit other symmetries of QCD, which are: Lorentz invariance, isosopin symmetry, parity (including intrinsic), hermiticity and charge conjugation invariance (CCI).

The first four symmetries can be implemented directly within the nonrelativistic formalism, as follows below. However, this is not obvious for charge conjugation invariance, because charge conjugation transformation relates the positive - and negative - energy sectors of the fermions. In this paper, there are no anti-nucleons, and hence the negative-energy field "h" (See (3)) has been integrated out of the nonrelativistic field theory. The new work has been in developing a method to ensure charge conjugation invariance directly in the framework of HBChPT, and is discussed in the next section.

(1) Lorentz invariance : Lorentz scalars (and pseudo-scalars) are constructed by contracting v_μ, D_μ (vectors, denoted by $\mathcal{V}_\mu$), S_μ, u_μ (axial-vectors, denoted by $\mathcal{A}_\mu$), χ_+ (scalar), χ_- (pseudo-scalar), with $\epsilon^{\mu\nu\rho\lambda}$ and $g_{\nu\rho}$. The discrete symmetry, parity, is used later to eliminate the pseudo-scalars. One considers the $\epsilon^{\mu\nu\rho\lambda}$-dependent and independent terms separately.

To ensure that all Lorentz indices have been contracted, the following condition has to be satisfied by the powers of the building blocks in (6):

$$(-)^{m+n+l+r} = 1. \tag{7}$$

(2) Isospin symmetry: Exact isospin symmetry is assumed, i.e. $m_u \neq m_d$ effects are ignored. The baryonic field is taken to be an iso-doublet: $\begin{pmatrix} p \\ n \end{pmatrix}$, and the mesonic field operators are constructed out of the field $\phi \equiv \vec{\pi} \cdot \vec{\tau}$ (and ∂_μ for the covariant derivative D_μ), where $\vec{\tau}$ are the Pauli nucleon isospin operators and $\vec{\pi}$ is the pion-isovector field. Terms in $\mathcal{L}_{(H)BChPT}$ are then isospin invariant.

(3) Parity : The parity operation is defined by $\vec{x} \to -\vec{x}, t \to t$ and $\phi \to -\phi$ (for pseudoscalar mesons), or in a covariant notation: $x_\mu \to x^\mu, \phi \to -\phi$.

One can show that u_μ, S_μ transform as pseudovectors, v_μ, D_μ transform as vectors and $\epsilon^{\mu\nu\rho\lambda}$ transforms as a pseudotensor. Hence, for terms of the form (6) including n u_μ's, q χ_-'s, r S_μ's and j tensors $\epsilon^{\mu\nu\rho\lambda}$ (with $j = 0$ or 1), a counting of the odd-parity terms such that

$$(-)^{n+q+r+j} = 1 \tag{8}$$

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makes the overall parity of the term even.

(4) Hermiticity : From the definitions of $D_\mu, u_\nu, \chi_\pm$ in (1), one can see that $(D_\mu^\dagger, \chi_-^\dagger) = -(D_\mu, \chi_-)$ and $(u_\mu^\dagger, \chi_+^\dagger) = (u_\mu, \chi_+)$. Now, if one uses a set of hermitian field operators : $(iD_\mu, u_\nu, \chi_+, i\chi_-)$, then one can define how to construct hermitian (anti-)commutators for all four field operators uniquely. Let

$$i^P(\mathcal{O}_1, \mathcal{O}_2) \equiv [\mathcal{O}_1, \mathcal{O}_2]_+, \text{ or } i[\mathcal{O}_1, \mathcal{O}_2]; \tag{9}$$

this is hermitian for hermitian building blocks $\mathcal{O}_{1,2}$, where P counts the number of commutators.

With the above considerations, one can consider hermiticity directly at the nonrelativistic level. (The nucleon building blocks v_μ and S_μ are hermitian.) But because we shall need to compare the hermiticity property with charge conjugation invariance in Section **3**, in the relativistic formalism, we return here to that formalism. Consider the following expression in BChPT:

$$i^P \bar{\psi} \Gamma \cdot (\mathcal{O}_1, (\mathcal{O}_2, (..., (\mathcal{O}_{n-1}, \mathcal{O}_n))....))\psi, \tag{10}$$

where $\Gamma \equiv$ any one of the 5 types of Dirac tensors. For Γ and $\mathcal{O}_i$, the phases h_Γ and $h_i(\equiv h_{\mathcal{O}_i})$ are defined via $\gamma^0 \Gamma^\dagger \gamma^0 = (-)^{h_\Gamma}\Gamma$ and $\mathcal{O}_i^\dagger = (-)^{h_i}\mathcal{O}_i = \mathcal{O}_i$. Taking the hermitian conjugate of (10), one gets:

$$\left(i^P \bar{\psi} \Gamma \cdot (\mathcal{O}_1, (\mathcal{O}_2, ...(\mathcal{O}_{n-1}, \mathcal{O}_n))....))\psi\right)^\dagger$$
$$= (-)^{h_\Gamma + P} i^P \bar{\psi} \Gamma \cdot ((...(\mathcal{O}_n, \mathcal{O}_{n-1}), \mathcal{O}_{n-2}), ...), \mathcal{O}_2), \mathcal{O}_1)\psi,$$
$$= (-)^{h_\Gamma} i^P \bar{\psi} \Gamma \cdot (\mathcal{O}_1, (\mathcal{O}_2, (....., (\mathcal{O}_{n-1}, \mathcal{O}_n))...))\psi. \tag{11}$$

3 Charge Conjugation Invariance

Unlike the other symmetries, charge conjugation invariance requires that one work in the relativistic formalism, and consequently CCI is defined in terms of BChPT. We show that CCI can be indirectly imposed in HBChPT, by requiring the invariance first in BChPT and then following its consequences into HBChPT. For that purpose, one needs to remember that v_μ, S_ν in HBChPT can always be obtained from $i[D_\mu,]_+, \gamma^5\gamma_\nu$ in BChPT.

Define the phases c_Γ and $c_{\mathcal{O}_i} \equiv c_i$ via $\mathcal{C}^{-1}\Gamma\mathcal{C} = (-)^{c_\Gamma}\Gamma^T$, and as we will see below, $\mathcal{O}_i^c = (-)^{c_i}\mathcal{O}_i^T$, where T is the transpose. Charge conjugating

(10), one gets:

$$\left(i^P \bar{\psi} \Gamma \cdot (\mathcal{O}_1, (..(\mathcal{O}_{n-1}, \mathcal{O}_n)..) \psi \right)^c = (-)^{c_\Gamma + c_\mathcal{O} + P} i^P \bar{\psi} \Gamma \cdot (\mathcal{O}_1, (..(\mathcal{O}_{n-1}, \mathcal{O}_n)..) \psi,$$

(12)

where $c_\mathcal{O} \equiv \sum_i^n c_i$.

One may obtain the charge conjugation properties of the four field opera-
tors from the QCD Lagrangian in the presence of external fields, assuming it
to be charge conjugation invariant. By doing so, one gets $i\mathrm{D}_\mu^c = -i\mathrm{D}_\mu^T, \mathrm{s}^c = \mathrm{s}^T, \mathrm{p}^c = \mathrm{p}^T$ (s, p are external scalar and pseudo-scalar fields). Now, as
$\phi \equiv \vec{\pi} \cdot \vec{\tau}$ transforms the same way under CCI as p, this implies $\phi^c = \phi^T$, which
in turn implies $u^c = u^T$, and hence $u_\mu^c = u_\mu^T$. Finally using $(s^c, p^c) = (s^T, p^T)$,
one gets $\chi_\pm^c = \chi_\pm^T$ (See (1) and footnote 1). One finds that $i\mathrm{D}_\mu$ is the only
charge-conjugate-odd building block.

Now, to ensure that CCI and hermiticity are satisfied simultaneously, the
following condition must obtain (comparing (11) and (12)):

$$(-)^{c_\Gamma + c_\mathcal{O} + P} = (-)^{h_\Gamma} = 1.$$

(13)

This equation remains unchanged if one considers expressions with traces.

Next, we show how this phase relation can be reexpressed in the non-
relativistic formalism. Because v_μ can be obtained from a non-relativistic
reduction of either $[i\mathrm{D}_\mu, \ ]_+$ or γ_μ, and S_ν from the nonrelativistic reduction
of $\gamma^5 \gamma_\nu$, it is in fact sufficient to consider only $\mathbf{1}, \gamma_\mu, \gamma^5 \gamma_\mu$ of the five types of
Dirac tensors. Further, one need not consider γ_μ, because any v_μ-dependent-
nonrelativistic term obtained from the reduction of a γ_μ - dependent - rela-
tivistic term, can also be obtained by a corresponding relativistic term with
the γ_μ replaced by $[i\mathrm{D}_\mu, \ ]_+$. (Note that γ_μ and $[i\mathrm{D}_\mu, \ ]_+$ have the same her-
miticity and charge conjugation properties). Also, as the hermiticity and
charge conjugation properties of $\mathbf{1}$ and $\gamma^5 \gamma_\mu$ are the same, S_μ does not affect
the phase in (13). So, finally it is sufficient to consider only $\Gamma \equiv \mathbf{1}$. Given
that $(-)^{c_\Gamma} = (-)^{h_\Gamma} = 1$, for $\Gamma \equiv \mathbf{1}$, (13) reduces to:

$$(-)^{c_\mathcal{O} + P} = 1.$$

(14)

We see from the discussion in the paragraph following (12) that $(-)^{c_\mathcal{O}}$
is equivalent to the phase factor coming from counting the number of $i\mathrm{D}_\mu$'s
in a given term in the BChPT Lagrangian. This in turn equals the phase

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factor coming from counting the number of v_μ's and iD_μ's in the equivalent HBChPT term obtained after taking the non-relativistic limit. Also, $(-)^P$ is equivalent to the number of commutators in the HBChPT term, which is the same as the number of commutators in the corresponding BChPT term. [Note that the number of anticommutators can change in the reduction from BChPT to HBChPT, since $[iD_\mu, \ ]_+$ in the former can produce v_μ in the latter. However, P remains unaffected.] So, one thus arrives at the following rule for constructing HBChPT terms that are hermitian and charge conjugation invariant:

Only those HBChPT terms are allowed, which consist of l v_μ's, m iD_ν's and $P[\ , \ ]$'s, and which are made hermitian using the prescription of (9), for which the following equation holds true:

$$(-)^{l+m+P} = 1. \tag{15}$$

It should be noted that (15) is an equation for HBChPT terms; no Dirac phases remain. Lorentz invariance and parity can be used for a further simplification. First, use (7) to rewrite (15) as

$$(-1)^{n+r+P} = 1 \tag{16}$$

Then, combining this with (8) yields the *phase rule* for charge conjugation invariant terms:

$$(-1)^{q+P+j} = 1. \tag{17}$$

This means that HBChPT terms consisting of q $i\chi_-$'s, $P[\ , \ ]$'s and j $(= 0$ or $1)$ $\epsilon^{\mu\nu\rho\lambda}$'s, that are Lorentz scalars and isoscalars of even parity, and are made hermitian using the prescription of (9), for which equation (17) is valid, are the only terms allowed. For application of (17), it is assumed one first writes down a complete list of hermitian Lorentz scalar-isoscalars satisfying chiral symmetry using the prescription of (9). The phase rule (17) is then used to pick out those HBChPT terms whose BChPT counterparts are also charge conjugation invariant.

We give some examples to illustrate the phase rule (17) in Table 2.

4 Reduction due to Algebraic Identities

In this section we use a variety of algebraic identities to reduce the number of independent terms in the Lagrangian, from that obtained by use of the

symmetry rules in the previous two sections.

4.1 Elimination of traces

For exact isospin symmetry, we may use several identities for traces on the isospin of the nucleons to show that trace-dependent terms may be eliminated from the Lagrangian, in favor of trace-independent terms. We classify field operators as isoscalar or isovector using the standard Pauli representation

$$\mathcal{O}_i = \mathcal{O}_i^0 \mathbf{1} + \mathcal{O}_i^a \tau^a \tag{18}$$

where $\mathbf{1} \equiv 2 \times 2$ unit matrix in the isospin space. Then $\mathcal{O}_i^a$ is the isoscalar component of $\mathcal{O}_i$ for $a = 0$, and is the isovector component (of $\mathcal{O}_i$) for $a = 1, 2, 3$. For example, χ_+ is isoscalar; χ_- and u_μ are isovectors. Then, if $\mathcal{O}$ is isoscalar or isovector,

$$\langle \mathcal{O} \rangle = 2\mathcal{O}, \tag{19}$$

or

$$\langle \mathcal{O} \rangle = 0. \tag{20}$$

These relations hold for $\mathcal{O}$ which are functions of the basic field operators, as well. The one combination of field operators which cannot appear within a trace is $[D_\mu, \mathcal{O}_j]_+$, since that would violate chiral symmetry (see Section **2**). In what follows, we exclude that operator from the $\mathcal{O}_i$. For all other operators, the coefficients of $\mathbf{1}$ or τ^a commute. Thus, the trace

$$\langle [\mathcal{O}_i, \mathcal{O}_j] \rangle = 0. \tag{21}$$

Similarly, it is easily shown, using (18) and the algebra of τ-matrices, that if $\mathcal{O}_i$ and $\mathcal{O}_j$ are both isoscalar, or both isovector,

$$\langle [\mathcal{O}_i, \mathcal{O}_j]_+ \rangle = 2[\mathcal{O}_i, \mathcal{O}_j]_+. \tag{22}$$

If one operator is isoscalar, and one isovector, then

$$\langle [\mathcal{O}_i, \mathcal{O}_j]_+ \rangle = 0. \tag{23}$$

As a result of (21) and (23), any trace-dependent term constructed from basic field operators can be put into one of the following four forms:

$$\langle [D_\mu, \mathcal{O}_1] \rangle; \ \langle [u_\nu, \mathcal{O}_2]_+ \rangle; \ \langle [\chi_+, \mathcal{O}_3]_+ \rangle; \ \langle [\chi_-, \mathcal{O}_4]_+ \rangle. \tag{24}$$

Now $D_\mu = \partial_\mu \mathbf{1} + \Gamma_\mu^a \tau^a$, where Γ_μ^a commutes with the other $\mathcal{O}_j$. It follows that for $\mathcal{O}_j$ isoscalar, that

$$\langle [D_\mu, \mathcal{O}_j] \rangle = 2[D_\mu, \mathcal{O}_j], \tag{25}$$

while for $\mathcal{O}_j$ isovector

$$\langle [D_\mu, \mathcal{O}_j] \rangle = 0. \tag{26}$$

We find that all the trace-dependent terms of either basic field operators, or of the forms of (21) or (24), can be reduced to equivalent forms without traces, using (19), (22) or (25), or else vanish, using (20), (21), (23) or (26). After this procedure, no trace-dependent terms remain. This result holds to all chiral orders.

4.2 Algebraic Reduction For Trace-Independent Terms

In this subsection, we introduce a number of algebraic identities which lead to linear relations among terms constructed from "building blocks," as in Sections **2** and **3**, and show how these may be used to reduce the number of independent terms of $O(q^3)$ in the Lagrangian. Since we can eliminate all trace-dependent terms, we discuss only trace-independent terms here.

First, the "curvature relation" (which holds in the absence of external vector and axial-vector fields) :

$$[D_\mu, D_\nu] = \frac{1}{4}[u_\mu, u_\nu], \tag{27}$$

is used extensively.

Second, we use the following three relations among commutators and/or anticommutators.

(i) If A, B are hermitian building-block field operators, then :

$$[A, [B, A]] \equiv 2ABA - [A^2, B]_+. \tag{28}$$

Sometimes for the purpose of comparison with EM's $O(q^3)$ list, one eliminates ABA and retains $[A, [B, A]], [A^2, B]_+$ ($A \equiv v \cdot u, B \equiv v \cdot D$ and $A \equiv u^\mu, B \equiv v \cdot D$).

(ii) Jacobi identity :

$$[A, [B, C]] = [[C, A], B] + [[A, B], C] \tag{29}$$

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(iii)

$$[A, [B, C]_+]_+ = ABC + ACB + \text{h.c.}$$
$$ABC + \text{h.c.} = -[[B, C], A] + BCA + \text{h.c.} \tag{30}$$

So, out of $[A, [B, C]_+]_+$, $ABC + \text{h.c.}$, $ACB + \text{h.c.}$, $[[B, C], A]$, one can take any two as linearly independent terms, say $ACB + \text{h.c.}$, $[[B, C], A]$.

The algebraic reductions associated with Levi-Civita (LC) independent terms are summarized in Table 3. (In Tables 3 and 4, the index i is used to indicate the $O(q^3)$ terms which have been selected for the final lists given in Section 5, in tables 7 and 8.)

For terms with the Levi-Civita tensor, we use the following relations.

$$\epsilon^{\mu\nu\rho\lambda} S_\lambda \Big([\mathcal{A}_\nu, [\mathcal{B}_\mu, \mathcal{C}_\rho]_+]$$
$$\equiv \epsilon^{\mu\nu\rho\lambda} S_\lambda \Big([\mathcal{B}_\mu, [\mathcal{A}_\nu, \mathcal{C}_\rho]]_+ + [[\mathcal{B}_\mu, \mathcal{A}_\nu], \mathcal{C}_\rho]_+ \Big), \tag{31}$$

where $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, are given by:

$$(a) \quad u_\nu, \ D_\mu, \ v_\rho v \cdot u,$$
$$(b) \quad v_\nu v \cdot u, \ D_\mu, \ u_\rho,$$
$$(c) \quad D_\nu, \ u_\mu, \ v_\rho v \cdot u,$$
$$(d) \quad D_\nu, \ u_\mu, \ u_\rho,$$
$$(e) \quad u_\nu, \ D_\mu, \ u_\rho. \tag{32}$$

From case (d), one finds that

$$\epsilon^{\mu\nu\rho\lambda} S_\lambda [u_\mu, [D_\nu, u_\rho]]_+ = 0, \tag{33}$$

from which it follows

$$\epsilon^{\mu\nu\rho\lambda} \bar{H} S_\lambda u_\nu D_\mu u_\rho H + \text{h.c.} = \frac{1}{2} \epsilon^{\mu\nu\rho\lambda} \bar{H} S_\lambda [u_\nu, [D_\mu, u_\rho]_+] H. \tag{34}$$

Finally, we replace (31) by the substitution $S_\lambda \to v_\lambda$, with $\mathcal{A}_\nu \equiv D_\nu$, $\mathcal{B}_\mu \equiv u_\mu$, $\mathcal{C}_\rho \equiv D_\rho$. Also:

$$i\epsilon^{\mu\nu\rho\lambda} \bar{H} v_\lambda D_\nu u_\mu D_\rho H + \text{h.c.}$$
$$= \frac{i}{2} \epsilon^{\mu\nu\rho\lambda} \bar{H} v_\lambda ([D_\nu, [u_\mu, D_\rho]_+] + [D_\nu, [u_\mu, D_\rho]]_+) H. \tag{35}$$

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The algebraic reductions associated with LC-dependent terms are summarized in Table 4.

Thus, using (27) - (35), one gets a large reduction in the trace-independent terms of O(q^3).

5   The Lists of Independent Terms in Off-Shell $\mathcal{L}_{\text{HBChPT}}$

In this section we give complete lists of terms in the Lagrangian of O(q^2), O(q^3) for off-shell nucleons. What appears in the following lists are terms in the Lagrangian of type $A^{(n)}$ (See (5) and [4]), for $n = 2, 3$, that can be written as sums of independent terms

$$\sum_{n=2}^{3} \sum_{i} \alpha_i^{(n)} (\mathcal{O}_i)^{(n)}, \tag{36}$$

where the low energy coupling constants (LECs) are given by the $\{\alpha_i^{(n)}\}$. (Note that $\{\alpha_i^{(2)}\} \equiv \{\text{``}a_i\text{''}\}$ in [6].)

For each of the nonrelativistic terms of the types $A^{(2)}$ and $A^{(3)}$, we will give the relativistic counterparts as well, for two reasons. First, comparison of the terms shows that there is a one-to-one correspondence between the chiral orders of the nonrelativistic terms and their relativistic counterparts. This implies that the coupling constants of the nonrelativistic terms of O(q^2) and O(q^3) are not fixed relative to those of O(q) and O(q, q^2), respectively. (This further implies that reparameterization invariance (RI), which we discuss in **7**, (See [10]) imposes no constraints for off-shell nucleons up to O(q^3)) Second, in the next section, we will consider the on-shell limit of the off-shell O(q^2) and O(q^3) terms (listed below in Table 5, 6, 7 and 8). The relativistic counterparts will help in this operation. For the purpose of comparison with the lists of Ecker and Mojzis [6] in Section **6**, the terms in these tables are categorized into three types: type $\mathcal{A}$ corresponding to the terms proportional to the nonrelativistic equation of motion; type $\mathcal{B}$ corresponding to those terms whose realtivistic counterparts can be rewritten as linear combinations of relativistic terms one of which is proportional to the relativistic equation of motion; terms present in [6] 's O(q^2) list (labeled by a$_i$) and O(q^3) list (labeled by b$_i$).

5.1 O(q^2) terms

In this subsection, we list terms in $A^{(2)}$ (Tables 5 and 6) separated into terms with ϕ^{2n} and ϕ^{2n+1}, i.e., even or odd powers of the pion field ϕ. (The same is done for $A^{(3)}$-type terms in the following subsection.)

One also gets some of the terms with fixed coefficients (i.e. independent of any LEC's other than axial-vector coupling constant g_A) that are $\frac{1}{m^0}$-suppressed from :

$$(\gamma^0 B^\dagger \gamma^0 C^{-1} B)^{(2)}. \tag{37}$$

(See [4])

5.2 O(q^3) terms

The algebraic reduction for trace-independent O(q^3) terms has been used to construct the tables 7 and 8. One also gets O(q^3) terms with O(q^2) LEC's that are $\frac{1}{m^{0^{1,2}}}$-suppressed (analogous to (37)) relative to $A^{(3)}$, from :

$$\left(\gamma^0 B^\dagger \gamma^0 C^{-1} B\right)^{(3)}. \tag{38}$$

We will not write these terms as we are interested in the number of independent O(q^3) LEC's that are required at O(q^3). It should be remembered that (37) and (38) $\in A^{(3)}$ for off-shell nucleons.

6 On-Shell Reduction

In this section we show the effect of putting the nucleons on-shell (i.e. the external nucleons are free). We have developed a method to obtain a complete on-shell reduction working entirely within HBChPT. The goal of this section is to give the rules within HBChPT for this on-shell reduction up to any chiral order and in particular of Tables 5, 6, 7 and 8 of **5**, for on-shell nucleons.

First in **6.1**, we derive a rule for carrying out on-shell reduction within HBChPT for the $A^{(2)}$ and $A^{(3)}$-type terms obtained in Sections **5.1**, **5.2**, and later for $A^{(n)}$ for all n. Next in **6.2**, we construct on-shell $B^{(n)}$ for all n, to evaluate the on-shell $\gamma^0 B^\dagger \gamma^0 C^{-1} B$ ("cross terms") of (5), within HBChPT. Specifically, we carry out the on-shell reduction of the O(q^3) cross terms. This will reduce the number of independent terms; the remaining terms so

obtained (through $O(q^3)$) are labelled by a_i in Table 6, and by b_i in Tables 7 and 8. These lists can be compared with similar lists given by Ecker and Mojzis (EM) [6], and can be shown to be completely equivalent. However, to get a complete on-shell reduction, EM first had to go back to $\mathcal{L}_{\text{BChPT}}$, and then take its nonrelativistic limit. Our method works directly in HBChPT. The match with EM's on-shell-reduced lists also serves as one check on the completeness of the off-shell lists (up to $O(q^3)$) obtained in Section **5**.

6.1   On-Shell Reduction of $A^{(n)}$

Taking the on-shell limit is equivalent to eliminating equation-of-motion (eom) terms in the Lagrangian. At the nonrelativisitic level, the A-type HBChPT terms obtained after $\frac{1}{m}$-reduction of BChPT eom terms get eliminated by a different technique, which is discussed in this subsection.

EM [6] have discussed how to eliminate the nonrelativistic eom terms by suitable field redefinition of H, up to $O(q^3)$. The ten terms of Tables 7 and 8 with $i = 11, 12, 13, 15, 16, 19, 22, 28, 29, 31$, are of this type, because they are explicitly proportional to the HBChPT eom (i.e. consist of $iv \cdot DH$). These terms are indicated by the label $\mathcal{A}$ in the last column, in Tables 7 and 8. We will not repeat the analysis of EM. However, it is not sufficient to eliminate all nonrelativistic eom terms within the framework of HBChPT, to obtain a complete reduction for on-shell nucleons. As an example, it can be shown that a class of HBChPT terms independent of $iv \cdot DH$, but consisting of $S \cdot DH$, can also be eliminated for on-shell nucleons [e.g.$i = 4$] (See (44)).

Krause [11] used an interesting technique to get a reduction in the terms for on-shell baryons in SU(3) BChPT, that we shall modify to suit the SU(2) case of this paper. We apply this method to the seven $A^{(3)}$-type terms with $i = 4, 9, 10, 14, 24, 26, 27$ (indicated by $\mathcal{B}$ in Tables 7 and 8). [We believe our method to be different from that of EM [6], who have taken the SU(2) limit of Krause's SU(3) list to ensure completeness.] All seven terms are such that after expanding the (anti-)commutators, they can be written as :

$$\bar{\text{H}}\mathcal{O}_\mu \text{D}^\mu \text{H} + \text{h.c.}, \tag{39}$$

which implies that for these terms the BChPT counterparts can be rewritten by "introducing" a term $(i\slashed{D} - \text{m}^0)\psi$, as will be shown below. This requires modifying Krause's technique to suit SU(2) (H)BChPT.

The basic idea is that a BChPT term written symbolically as:

$$\bar{\psi}(\Gamma\mathcal{O})_\mu D^\mu \psi \tag{40}$$

(with $\Gamma \equiv$ fundamental Dirac tensor, $\mathcal{O} \equiv$ product of building blocks, coupled to $(\Gamma\mathcal{O})_\mu$) can be rewritten as:

$$\begin{aligned}
\bar{\psi}(\Gamma\mathcal{O})_\mu g^{\mu\nu} D_\nu \psi &\equiv \bar{\psi}(\Gamma\mathcal{O})_\mu(\gamma^\mu\gamma^\nu + i\sigma^{\mu\nu})D_\nu\psi \\
&\equiv -i\bar{\psi}(\Gamma\mathcal{O})_\mu\gamma^\mu(i\slashed{D} - m^0)\psi - im^0\bar{\psi}(\Gamma\mathcal{O})_\mu\gamma^\mu\psi \\
&+i\bar{\psi}(\Gamma\mathcal{O})_\mu\sigma^{\mu\nu}D_\nu\psi.
\end{aligned} \tag{41}$$

Writing $\Gamma\sigma^{\mu\nu} = \sum_{i=1}^5 a_i\Gamma_i$ (for $i \equiv$ S, PS, V, A, T) from the completeness of the Γ_i's, except for $\Gamma \equiv \mathbf{1}$, γ^5, [2] there must be an $i = j$ such that the tensor type of Γ is the same as that of Γ_j. There are then two possiblities: (a) the Lorentz indices of Γ and Γ_j are the same, or (b) the Lorentz indices of Γ and Γ_j are different.

(a) Lorentz indices of Γ and Γ_j are the same:

Rewriting (41) one obtains:

$$\begin{aligned}
(1 - a_j)\bar{\psi}(\Gamma\mathcal{O})_\mu D^\mu\psi &= -i\bar{\psi}(\Gamma\mathcal{O})_\mu\gamma^\mu(i\slashed{D} - m^0)\psi - im^0\bar{\psi}(\Gamma\mathcal{O})_\mu\gamma^\mu\psi \\
&+ \sum_{i \neq j}\bar{\psi}(a_i\Gamma_i\mathcal{O})_\mu D^\mu\psi.
\end{aligned} \tag{42}$$

Since the first term on the RHS of (42) is proportional to the BChPT eom, it can be eliminated by suitable field redefinition of ψ. The second term (on the RHS of (42)) is of a lower order than the original term, and hence can be discarded. The third term remains.

Now, as long as $a_j \neq 1$, one sees that the $\bar{\psi}(\Gamma\mathcal{O})_\mu D^\mu\psi$ can be rewritten as a linear combination of terms distinct from itself. Thus, the nonrelativistic term obtained from the $\frac{1}{m}$-reduction of LHS of (42) can be written as linear combination of other nonrelativistic terms obtained from $\frac{1}{m}$-reduction of RHS of (42) which are already present in the complete list of terms. (We find $a_j \neq 1$ to the case for $i = 9, 14, 24, 26, 27$.) For the case $a_j = 1$, (42) implies that nonrelativistic terms other than (39), obtained from the $\frac{1}{m}$-reduction of

---

[2]For $\Gamma \equiv \mathbf{1}$, γ^5, the analysis is the same as the one following (42); the difference is that instead of (42), it is (41) that should be referred to in the analysis. The choice $\Gamma \equiv \gamma^5$ is relevant for constructing on-shell B. (See **6.2**)

(42), are going to be related. As will be shown later, there is only one term for which one gets $a_j = 1$ in (42): $i = 21$.

(b) Lorentz indices of Γ and Γ_j are different (as is the case for $i = 10$):

$$
\begin{aligned}
\bar{\psi}(\Gamma\mathcal{O})_\mu \mathrm{D}^\mu \psi &= -i\bar{\psi}(\Gamma\mathcal{O})_\mu \gamma^\mu (i\slashed{D} - \mathrm{m}^0)\psi - i\mathrm{m}^0 \bar{\psi}(\Gamma\mathcal{O})_\mu \gamma^\mu \psi \\
&\quad + \sum_i \bar{\psi}(a_i\Gamma_i\mathcal{O})_\mu \mathrm{D}_\nu \psi.
\end{aligned} \tag{43}
$$

The implications are exactly the same as that of (42) (for $a_j \neq 1$), except that the LHS of (43) cannot be obtained from the third term on the RHS of (43).

The analysis of (41) gets simplified if the $\mathcal{O}_\mu$ in (39) can be expressed as $\mathrm{S}_\mu \mathcal{O}(\mathcal{O} \equiv$ building block). Consider the term $\bar{\mathrm{H}}\mathrm{S} \cdot \mathrm{D}\mathcal{O}\mathrm{H}$+h.c. Its relativistic counterpart is $\bar{\psi}\gamma^5 \slashed{D}\mathcal{O}\psi$+h.c., which can be rewritten as:

$$
\begin{aligned}
&\bar{\psi}\gamma^5 \slashed{D}\mathcal{O}\psi + \mathrm{h.c.} \\
&\equiv i\bar{\psi}(-i\overleftarrow{\slashed{D}} - \mathrm{m})\gamma^5\mathcal{O}\psi + i\mathrm{m}\bar{\psi}\gamma^5\mathcal{O}\psi + \mathrm{h.c.}.
\end{aligned} \tag{44}
$$

The first term can be eliminated by a redefinition of ψ. The second term is not going to reduce to the required nonrelativistic term; hence $\bar{\mathrm{H}}\mathrm{S}\cdot\mathrm{D}\mathcal{O}\mathrm{H}$+h.c. can be eliminated for on-shell nucleons. (This eliminates the $i = 4$ term.)

Using (41) - (44), one arrives at a rule directly within the nonrelativistic formalism for elimination of those $A^{(n)}$, $n = 2, 3$ terms that are either of the type $\bar{\mathrm{H}}\mathrm{S}\cdot\mathrm{D}\mathcal{O}\mathrm{H} + \mathrm{h.c.}$ or are such that their relativistic counterparts can be rewritten as a linear combination of other relativistic terms one of which is the BChPT eom:

$$
\begin{aligned}
&\text{HBChPT terms of the type } \bar{\mathrm{H}}\mathrm{S}\cdot\mathrm{D}\mathcal{O}\mathrm{H} + \mathrm{h.c.} \\
&\text{or } \bar{\mathrm{H}}\mathcal{O}^\mu \mathrm{D}_\mu \mathrm{H} + \mathrm{h.c. \ can \ be \ eliminated} \\
&\text{except for } \mathcal{O}_\mu \equiv iu_\mu v\cdot u.
\end{aligned} \tag{45}
$$

It is understood that all (anti-)commutators in the HBChPT Lagrangian are to be expanded out until one hits the first D_μ, so that the A-type HBChPT term can be put in the form $\bar{\mathrm{H}}\mathcal{O}^\mu \mathrm{D}_\mu \mathrm{H}$ +h.c.

The $A^{(2)}$-type terms $i = 1, 2, 6, 7$, can be shown to be eliminated for on-shell nucleons by application of the abovementioned rule. Then one finds agreement between the remaining terms in Tables 5 and 6 with EM's list of

$A^{(2)}$ terms, as indicated in the last column, where a_i labels the term as in EM. Similarly, the $A^{(3)}$ terms already discussed can be eliminated for on-shell nucleons, and are so denoted in Tables 7 and 8. The remaining terms appear in the EM list, where b_i indicates their terms. With the addition of the exceptional term $i = 21$, discussed next, the agreement of our construction with that of EM of $A^{(n)}$ through $O(q^3)$ is complete.

The reason why $\mathcal{O}_\mu = iu_\mu v \cdot u$ is an exception to (45) has to do with the fact that $\Gamma_\mu = \gamma_\mu$ in (40), as explained below. The relativistic counterpart of the $i = 21$ term: $i\bar{H}u^\mu v \cdot u D_\mu H + $ h.c. is:

$$\bar{\psi}(iu^\mu \slashed{u} D_\mu + \text{h.c.})$$
$$= \bar{\psi}u^\mu \slashed{u}\gamma_\mu(i\slashed{D} - m^0)\psi + m^0\bar{\psi}u^\mu \slashed{u}\gamma_\mu\psi - \bar{\psi}\gamma_\rho\sigma_{\mu\nu}u^\mu u^\rho D^\nu\psi + \text{h.c.} \quad (46)$$

After some algebra, one gets:

$$\bar{\psi}\left(i[\slashed{u}, u^\mu]D_\mu + \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}\gamma^5\gamma_\lambda[[u_\mu, u_\rho], D_\nu]_+\right)\psi$$
$$= \bar{\psi}\left(\frac{1}{2}\sigma^{\mu\nu}[u_\mu, u_\nu](i\slashed{D} - m^0) + \frac{1}{2}m^0\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\psi. \quad (47)$$

Thus, $i = 21$ term drops out from the above relativistic equation (and is therefore not eliminated as an eom term), which is equivalent to setting $a_j = 1$ in (42). The second and the first terms on the LHS of (47) (after $\frac{1}{m}$-reduction) give the $i = 27$ term, and a term $i\bar{H}[[v \cdot u, u^\mu], D_\mu]H$ which can be eliminated for off-shell nucleons (See Table 3). (Further, by using (45), the $i = 27$ term can also be eliminated for on-shell nucleons.)

To generalise, one sees that if in (40), $\Gamma_\mu \equiv \gamma_\mu$, then by the application of (41), that relativistic term, and hence its nonrelativistic counterpart, will not be eliminated for on-shell nucleons. Now, γ_μ gives v_μ after $\frac{1}{m}$-reduction, which can enter the HBChPT Lagrangian only as $v \cdot D$ or $v \cdot u$ or $\epsilon^{\mu\nu\rho\lambda}v_\rho$. A term with $v \cdot D$ can be rewritten as linear combinations of terms one (or more) of which has a $v \cdot DH$ that can be eliminated by field redefinition of H. The remaining terms in the linear combination would either have already been included, or can be considered as separate terms which can be obtained independently. Now, $\epsilon^{\mu\nu\rho\lambda}v_\rho(v \cdot u)$ can be obtained from the nonrelativistic reduction of $\epsilon^{\mu\nu\rho\lambda}i[D_\rho,]_+\slashed{u}$. Also, because $D^2 + m^2$ after $\frac{1}{m}$-reduction gives $(D^2 - 2imv \cdot D)$ and that $\epsilon^{\mu\nu\rho\lambda}v_\rho(v \cdot u)^{l_1}(l_1 \geq 1)$ can be obtained from the nonrelativistic reduction of $\epsilon^{\mu\nu\rho\lambda}i[D_\rho,]_+\slashed{u}(iu \cdot D)^{l_1-1}$, one sees that the following class of nonrelativistic terms will also serve as the exceptions to the

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above rule if one is to extend it to higher orders (i.e. beyond third order):

$$\mathcal{O}_\mu \equiv \left(i^{m_1+l_5+l_7+1}, \text{ or } \epsilon^{\nu\lambda\kappa\rho} \times \Omega \right) \times u_\mu \Lambda \tag{48}$$

with $l_1 \geq 1$, $\Omega \equiv 1(i)$ for $(-)^{m_1+l_5+l_7+1} = -1(1)$, or

$$\mathcal{O}_\mu \equiv \left(i^{m_1+l_5+l_7}, \text{ or } \epsilon^{\nu\lambda\kappa\rho} \times \Omega' \right) \times D_\mu \Lambda, \tag{49}$$

with $l_1 \geq 1$, $\Omega' \equiv 1(i)$ for $(-)^{m_1+l_5+l_7} = -1(1)$. In (48) and (49),

$$\Lambda \equiv \prod_{i=1}^{M_1} \mathcal{V}_{\nu_i} \prod_{j=1}^{M_2} u_{\rho_j} (v \cdot u)^{l_1} u^{2l_2} \chi_+^{l_3} \chi_-^{l_4} ([v \cdot D, \ ])^{l_5} (D_\beta D^\beta)^{l_6} (u_\alpha D^\alpha)^{l_7}, \tag{50}$$

where $\mathcal{V}_{\nu_i} \equiv v_{\nu_i}$ or D_{ν_i}. The number of D_{ν_i}s in (50) (and in (51), below) equals $m_1 (\leq M_1)$. Assuming that Lorentz invariance, isospin symmetry, parity and hermiticity have been implemented, the choice of the factors of i in (48) - (50) (and (51)) automatically incorporates the phase rule (17). In (48) - (50) (and (51)), it is only the contractions of the building blocks that has been indicated. [3] It is understood that $v \cdot D$ always enters as a commutator with another building block. (The $i=21$ term can be obtained from (48) as a particular case.) On comparing (48) - (50) with (40), $\Gamma_\mu \equiv \gamma_\mu$. Thus, just as for $i = 21$ term, (48) - (50) will not be eliminated for on-shell nucleons by the application of (45).

6.2 On-Shell Reduction of $B^{(n)}$

For on-shell nucleons, the "cross terms" of (5) are not contained in A, and to carry out a complete on-shell reduction, one has to use (5); the phase rule (17) (used for construction of A) will not suffice. As an example, it is possible that for on-shell nucleons, one may be able to eliminate [as an eom term, the form] S·D$\mathcal{O}$ that one would get from $A^{(n)}$-type terms, but one may still get the same

---

[3]For L.C-(in)dependent terms in (48) - (50) with $l_7(>) \geq 1$, the coupling constants will be fixed relative to those of lower order terms, which is a consequence of reparameterization invariance. In this discussion however (*unlike* **5**), we are interested in the number of independent terms, and not the number of independent coupling constants.

term from $\left(\gamma^0 B^\dagger \gamma^0 C^{-1} B\right)^{(n)}$ ("cross terms") in (5), where $B^{(n-1)} = \gamma^5 \mathcal{O}$ and $B^{(1)} = i\gamma^5 \mathrm{S\cdot D}$. This is because of the following. The abovementioned $B^{(n-1)}$ comes from the nonrelativistic reduction of $\bar{\psi}\gamma^5 \mathcal{O}\psi$, (and $B^{(1)}$ comes from the Dirac term $\bar{\psi}(i\slashed{D} - \mathrm{m})\psi$). Because γ^5, after $\frac{1}{\mathrm{m}}$-reduction contributes only to B and not to A, and because for on-shell nucleons, the cross terms can not always also be obtained from the A-type terms, it thus becomes necessary to take the on-shell limit of $B^{(n-1)}$-type terms (which contribute at $\mathrm{O}(q^n)$) in addition to taking the on-shell limit of $A^{(n)}$.

Thus, in order to construct the cross terms within HBChPT, one should be able to construct on-shell B within HBChPT.

Using arguments similar to those used in **6.1**, one sees that the following gives the on-shell B:

$$
B(\mathrm{on - shell}) \equiv \Gamma_B \times \left(i^{m_1 + l_5 + l_7}, \text{ or } \epsilon^{\nu\lambda\kappa\rho} \times \Omega''\right)
$$
$$
\times \prod_{i=1}^{M_1}\left(v_{\nu_i} \text{ or } [\mathrm{D}_{\nu_i}, \ ]\right) \prod_{j=1}^{M_2} u_{\rho_j}
$$
$$
\times (v \cdot u)^{l_1} u^{2l_2} \chi_+^{l_3} \chi_-^{l_4} ([v \cdot \mathrm{D}, \ ])^{l_5} ([\mathrm{D}_\beta, \ ])^{2l_6} ([\mathrm{D}^\alpha, \ ]u_\alpha)^{l_7},
$$
$$(51)$$

where $(-)^{M_2 + l_1 + l_4 + l_7} = (1)-1$ for L.C.-(in)dependent (51) if $\Gamma_B \equiv \gamma^5 \mathrm{S}_\mu, \gamma^5 v_{[\mu}\mathrm{S}_{\nu]}$, and $(-)^{M_2 + l_1 + l_4 + l_7} = (-1)1$ for L.C.-(in)dependent (51) if $\Gamma_B \equiv \gamma^5, \gamma^5 v_\mu$, $\epsilon^{\mu\omega\sigma\delta}\gamma^5 v_\sigma \mathrm{S}_\delta$ [4] (for even parity of B). [5] Also, $\Omega'' \equiv 1(i)$ if $(-)^{m_1 + l_5 + l_7} = 1(-1)$ for L.C.-independent terms, and $\Omega'' \equiv 1(i)$ if $(-)^{m_1 + l_5 + l_7} = -1(1)$ for L.C.-dependent terms. It is understood that $v \cdot \mathrm{D}$, $\mathrm{D}_{\alpha,\beta,\nu_i}$ always enter as a commutator in (51), so that $v \cdot \mathrm{D}$ and $\mathrm{D}_{\alpha,\beta,\nu_i}$ can not act on H; the requirement of $\mathrm{D}_{\alpha,\beta,\nu_i}$ entering only as a commutator can be relaxed only for $\Gamma_B \equiv \gamma^5 \mathrm{S}^\mu$ (See **6.1** and footnote 4). It is however, still possible in some cases to rewrite (51) as:

$$
(\Gamma_B \mathcal{O}')^\mu \mathrm{D}_\mu. \tag{52}
$$

---

[4] $\epsilon^{\nu\lambda\kappa\rho} \times (\Gamma_B \equiv \epsilon^{\mu\omega\sigma\delta}\gamma^5 v_\sigma \mathrm{S}_\delta) \times (......)$ in (51) is to be treated as L.C-independent, because product of even number of L.C.'s can be written as products of metric.

[5] Use is made of that $\bar{\mathrm{h}}\left(\gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, \sigma^{\mu\nu}, \gamma^5\sigma^{\mu\nu}\right)\mathrm{H}$ gives $\bar{\mathrm{h}}\left(\gamma^5, \gamma^5\mathrm{S}^\mu, \gamma^5 v^\mu, i\gamma^5 v^{[\mu}\mathrm{S}^{\nu]}, \epsilon^{\mu\nu\rho\lambda}\gamma^5 v_\rho \mathrm{S}_\lambda\right)\mathrm{H}$ (omitting factors of 2).

Such terms are to be excluded in constructing on-shell B. Thus, one need not consider $i\gamma^5[\mathrm{D}_\mu, u^\mu] \in B^{(2)}$.

One can show that the contribution of the on-shell limit of $B^{(2)}$, gives one additional term: $i = 6 :\ i\bar{\mathrm{H}}[\mathrm{S} \cdot \mathrm{D}, \chi_-]\mathrm{H};\ \gamma^5\chi_-$ can be obtained as a special case of (51). One should thus note that for on-shell nucleons, the LEC of the $i = 6$ term is that of the term $\gamma^5\chi_- \in B^{(2)}$. With the addition of $i = 6$ term, the agreement of our construction with that of [6] of on-shell (5) through $\mathrm{O}(q^3)$ is complete.

So, terms of the form (51) appear in the on-shell list of B unless they can be rewritten as (52). Combining the conclusions of **6**.**1** and **6**.**2**, equations (45), (48) - (52) [modulo algebraic reductions] give rules for the construction of the Lagrangian (5) on-shell, within HBChPT.

7   Conclusion and Discussion

The goal of this paper is to develop a method to generate a complete expansion of $\mathcal{L}_{\mathrm{HBChPT}}$ to a given order directly within the framework of HBChPT. A new method has been developed to implement CCI (combined with other symmetries discussed in **2**) within the framework of HBChPT, resulting in a phase rule derived in the paper. Additional reduction of the number of independent terms, using algebraic identities was subsequently carried out, which, incidentally, allows the omission of traces (for exact isospin symmetry). The method was applied to generate complete lists of $A^{(2),(3)}$ for off-shell nucleons. We obtain 8 $\mathrm{O}(q^2)$ and 31 $\mathrm{O}(q^3)$ independent terms, with undetermined low energy coupling constants (LECs). The method has also been carried out to next order, for terms of $\mathrm{O}(q^4, \phi^{2n})$, to be discussed in a future publication.

The main advantage of the method developed in the paper for constructing the off-shell $\mathcal{L}_{\mathrm{HBChPT}}$, as compared to the standard $\frac{1}{\mathrm{m}}$-reduction formalism in the literature [4], [6], is the following. Unlike the standard $\frac{1}{\mathrm{m}}$-reduction formalism, one is not required to know (and therefore to construct) the exact form of $\mathcal{L}_{\mathrm{BChPT}}$, and then to perform the non-relativistic term-by-term reduction of $\mathcal{L}_{\mathrm{BChPT}}$, in order to construct $\mathcal{L}_{\mathrm{HBChPT}}$. Thus, the phase rule method of this paper is more efficient than the standard $\frac{1}{\mathrm{m}}$-reduction formalism in the literature, for off-shell nucleons. In the present method, one-to-one correspondence between (the chiral orders of) $\mathcal{L}_{\mathrm{HBChPT}}$ and its counterpart

$\mathcal{L}_{\mathrm{BChPT}}$, is automatically implied. We have verified this in detail through $O(q^3)$, but there are exceptions, e.g. in certain terms of $O(q^4, \phi^{2n})$, to be discussed in a future publication. The exceptions arise due to reparameterization invariance (RI) according to which the Lagrangian density is invariant under infinitesimal variations of the nucleon-velocity parameter. As a consequence, the coupling constants of certain terms at a given chiral order get fixed relative to those of some terms at lower chiral orders.

Also, a method is given for obtaining the on-shell terms of types A and B, and hence the HBChPT Lagrangian, entirely within HBChPT. For the purpose of comparison with EM's complete lists of $A^{(2),(3)}$-type terms for on-shell nucleons [6], the lists of $A^{(2),(3)}$-type terms obtained in Section **5** were reduced within HBChPT, after elimination of 'equation of motion' (eom) terms (both at the nonrelativistic and relativistic levels), using a rule developed in Section **6** for arbitrary chiral orders. After performing the on-shell reduction up to $O(q^3)$, agreement was found with the lists given in the EM paper. By developing a method of constructing the on-shell B (within HBChPT), the issue of imposing RI directly within HBChPT gets partially addressed in the sense that the LECs of some $O(q^n)$ terms get fixed relative to the LECs of some $O(q^{n-1})$ B-type terms. (E.g., the LECs of the nucleon kinetic energy and the $O(q^2)$-correction to the Yukawa term which are D^2 and $i[\mathrm{S}\cdot\mathrm{D}, v\cdot u]_+$ respectively, get fixed relative to the Dirac and Yukawa terms respectively.) However, the RI constraints on A-type terms in higher order than $O(q^3)$ will still need to be worked out (see footnote 2); this is beyond the scope of this paper.

Acknowledgements

References

[1] J.Gasser and H.Leutwyler, Annals of Physics 158 (1984) 142.

[2] J.Gasser, M.E.Sainio and A.Svarc, Nucl. Phys.B 307 (1988) 779.

[3] E.Jenkins and A.V.Manohar, Phys. Lett. B 255 (1991) 558.

[4] V.Bernard, N.Kaiser, J.Kambor and Ulf-G Meissner, Nucl.Phys.B 388 (1992) 315.

[5] J.G.Koerner J.G.Thompson, Phys. Lett. B 264 (1991) 185.

[6] G.Ecker and M.Mojzis, Phys. Lett. B 365 (1996) 312.

[7] G.Ecker, J.Gasser, A.Pich and E. de Rafael, Nucl. Phys. B 321 (1989) 311.

[8] A.Misra, Ph.D. Thesis,
$SU(2)$ *Heavy Baryon Chiral Perturbation Theory for One − and Two − Nucleon Processes*; *Application to Pion Double Charge Exchange to One Loop*, University of Rochester, 1997, Report UR 1508.

[9] T.Mannel, W.Roberts. W.Ryzak, Nucl. Phys. B 368 (1992) 204.

[10] M.Luke and A.V.Manohar Phys. Lett.B 286 (1992) 348.

[11] A.Krause, Helv. Phys. Acta 63 (1990) 3.

Table 1: Allowed 4-tuples for $O(q^3)$ terms

$O(q^3, \phi^{2n})$		$O(q^3, \phi^{2n+1})$	
(3,0,0,0)	(1,2,0,0)	(0,3,0,0)	(2,1,0,0)
(1,0,1,0)	(0,1,0,1)	(0,1,1,0)	(1,0,0,1)

Table 2: Examples of application of the phase rule (17)

HBChPT term	$(-)^q$	$(-)^P$	$(-)^j$	$(-)^{q+P+j}$	Y $\equiv$ allowed N $\equiv$ not allowed
$\epsilon^{\mu\nu\rho\lambda} v_\mu i[i\mathrm{D}_\mu, [i\mathrm{D}_\rho, u_\lambda]_+]$	$+$	$-$	$-$	$+$	Y
$\epsilon^{\mu\nu\rho\lambda} \mathrm{S}_\mu [\mathrm{u}_\nu, \mathrm{i}[\mathrm{u}_\rho, \mathrm{iD}_\lambda]_+]_+$	$+$	$+$	$-$	$-$	N
$[v \cdot u, i[i\mathrm{D}^\mu, u_\mu]]_+$	$+$	$-$	$+$	$-$	N
$[\mathrm{S} \cdot u, i[i\chi_-, [i\mathrm{D}^\mu, u_\mu]_+]]_+$	$-$	$-$	$+$	$+$	Y

Table 3: Algebraic reduction in L-C-independent terms

Need not consider	If consider	Equations
$[u_\mu, [\mathrm{D}^\mu, \mathrm{S}\cdot\mathrm{D}]_+]_+$; $u\cdot\mathrm{DS}\cdot\mathrm{D} + \mathrm{h.c.}$; $u_\mu\mathrm{S}\cdot\mathrm{DD}^\mu + \mathrm{h.c.}$; $[\mathrm{D}^\mu, \mathrm{S}\cdot\mathrm{D}], u_\mu]$; $[\mathrm{D}_\mu, [u^\mu, \mathrm{S}\cdot\mathrm{D}]_+]_+$; $\mathrm{D}\cdot u\mathrm{S}\cdot\mathrm{D} + \mathrm{h.c.}$; $[[u^\mu, \mathrm{S}\cdot\mathrm{D}], \mathrm{D}_\mu]$; $[[\mathrm{S}\cdot u, u^\mu], u_\mu]$; $[[\mathrm{D}_\mu, u^\mu]_+, \mathrm{S}\cdot\mathrm{D}]_+$	$i = 1, 2, 4$	(27)-(30)
Same as above with $u_\mu \to v\cdot u$; $\mathrm{D}_\mu \to v\cdot\mathrm{D}$	$i = 7, 8, 12$	(27) - (30)
$[\mathrm{D}_\mu, [u^\mu, v\cdot u]_+]_+$; $i\mathrm{D}\cdot uv\cdot u + \mathrm{h.c.}$; $i[[u^\mu, v\cdot u], \mathrm{D}_\mu]$: $i[[v\cdot u, \mathrm{D}_\mu], u^\mu]$; $i[[\mathrm{D}_\mu, u^\mu], v\cdot u]$; $iu\cdot\mathrm{D}v\cdot\mathrm{u} + \mathrm{h.c.}$; $i[v\cdot u, [u^\mu, \mathrm{D}_\mu]_+]_+$; $\mathrm{i}[\mathrm{D}_\mu, [\mathrm{D}^\mu, v\cdot\mathrm{D}]]$; $i\mathrm{D}_\mu v\cdot\mathrm{DD}^\mu$	$i = 21, 23, 29$	(27)-(30)
$i(v\cdot u)(v\cdot\mathrm{D})(v\cdot u)$	$i = 16, 17$	(28)
$iu_\mu(v\cdot\mathrm{D})u^\mu$	$i = 15, 18$	(28)

Table 4: Algebraic reductions of L-C-dependent terms

Need not consider	If consider	Equations
$\epsilon^{\mu\nu\rho\lambda}v_\rho\mathrm{S}_\lambda\Big([u_\nu, [\mathrm{D}_\mu, v\cdot u]_+]$; $[v\cdot u, [\mathrm{D}_\mu, u_\nu]_+]$; $[[\mathrm{D}_\mu, v\cdot u], u_\nu]_+$; $[\mathrm{D}_\mu, [\mathrm{D}_\nu, v\cdot\mathrm{D}]]_+$; $[\mathrm{D}_\mu, [u_\nu, v\cdot u]_+]\Big)$	$i = 24, 25, 26$	(27), (31), (32) [(a)-(c)]
$\epsilon^{\mu\nu\rho\lambda}\mathrm{S}_\lambda\Big([u_\nu, [\mathrm{D}_\mu, u_\rho]_+]$; $u_\nu\mathrm{D}_\mu u_\rho$; $[\mathrm{D}_\mu, [\mathrm{D}_\nu, \mathrm{D}_\rho]]_+\Big)$	$i = 27$	(27), (31), (32)[(e)], (33), (34)
$i\epsilon^{\mu\nu\rho\lambda}v_\lambda\Big([\mathrm{D}_\nu, [u_\mu, \mathrm{D}_\rho]_+]$; $[[u_\mu, \mathrm{D}_\nu], \mathrm{D}_\rho]_+$; $[u_\mu, [\mathrm{D}_\nu, \mathrm{D}_\rho]]_+\Big)$	$i = 3, 14$	(27), (31)[$\mathrm{S}_\lambda \to v_\lambda$] (35)

Table 5: $A^{(2)}(\phi^{2n+1})$ terms

i	$A^{(2)}(\phi^{2n+1})$	BChPT Counterparts	Term type: $\mathcal{A}$, $\mathcal{B}$; a_i in EM
1	$i[\text{S}\cdot\text{D}, v\cdot u]_+$	$\frac{1}{\text{m}^0}\gamma^5[\slashed{D}, [u^\mu, \text{D}_\mu]_+]_+$	$\mathcal{B}$
2	$i[\text{S}\cdot u, v\cdot\text{D}]_+$	$\frac{1}{\text{m}^0}\gamma^5\left[\slashed{u}, \left(\text{D}^2+\text{m}^2-\left(-(i\slashed{D}-\text{m}^0)^2 +\frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right)\right]_+$	$\mathcal{A}$

Table 6: $A^{(2)}(\phi^{2n})$ terms

i	$A^{(2)}(\phi^{2n})$	BChPT Counterparts	Term type: $\mathcal{A}$, $\mathcal{B}$; a_i in EM
3	$(v\cdot u)^2$	$\frac{1}{\text{m}^{0^2}}[\text{D}_\mu, u^\mu]_+^2$	a_2
4	u^2	u^2	a_1
5	$[\text{S}_\mu, \text{S}_\nu][u^\mu, u^\nu]$	$i\sigma^{\mu\nu}[u_\mu, u_\nu]$	a_5
6	$(v\cdot\text{D})^2$	$\frac{1}{\text{m}^{0^2}}\left(\text{D}^2+\text{m}^2-\left(-(i\slashed{D}-\text{m}^0)^2+\frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right)^2$	$\mathcal{A}$
7	D^2	$\left(-(i\slashed{D}-\text{m}^0)^2+\frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)$	$\mathcal{B}$
8	χ_+	χ_+	a_3

i	$A^{(3)}(\phi^{2n+1})$	BChPT Counterparts	Term type: $\mathcal{A}$, $\mathcal{B}$; b_i in EM
1	$[u^2, \mathrm{S}\cdot u]_+$	$\gamma^5[u^2, \slashed{u}]_+$	b_{11}
2	$u^\mu \mathrm{S}\cdot u u_\mu$	$\gamma^5 u^\mu \slashed{u} u_\mu$	b_{12}
3	$i\epsilon^{\mu\nu\rho\lambda} v_\mu[[u_\nu, u_\rho], u_\lambda]_+$	$i\epsilon^{\mu\nu\rho\lambda}\gamma_\mu[[u_\nu, u_\rho], u_\lambda]_+$	b_5
4	$[\mathrm{S}\cdot\mathrm{D}, [\mathrm{D}^\mu, u_\mu]]$	$\gamma^5[\slashed{D}, [\mathrm{D}_\mu, u^\mu]]$	$\mathcal{B}$
5	$[\mathrm{S}\cdot u, \chi_+]_+$	$[\gamma^5\slashed{u}, \chi_+]_+$	$b_{17,18}$
6	$i[\mathrm{S}\cdot\mathrm{D}, \chi_-]$	$i\gamma^5[\slashed{D}, \chi_-]$	b_{19}
7	$v\cdot u \mathrm{S}\cdot u v\cdot u$	$\frac{1}{\mathrm{m}^{0^2}}\gamma^5[\mathrm{D}_\mu, u^\mu]_+\slashed{u}[\mathrm{D}_\nu, u^\nu]_+$	b_{14}
8	$[\mathrm{S}\cdot u, (v\cdot u)^2]_+$	$\frac{1}{\mathrm{m}^{0^2}}\gamma^5[\slashed{u}, [[\mathrm{D}_\mu, u^\mu]_+, [\mathrm{D}_\nu, u^\nu]_+]_+]_+$	b_{13}
9	$[\mathrm{S}\cdot u, \mathrm{D}^2]_+$	$\gamma^5\left[\slashed{u}, \left(-(i\slashed{D}-\mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right]_+$	$\mathcal{B}$
10	$\mathrm{D}^\mu \mathrm{S}\cdot u \mathrm{D}_\mu$	$\gamma^5\mathrm{D}_\mu\slashed{u}\mathrm{D}^\mu - \frac{1}{2}\gamma^5\left[\left(\mathrm{D}^2 + \mathrm{m}^{0^2} -\left(-(i\slashed{D}-\mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right), \slashed{u}\right]_+$	$\mathcal{B}$

i	$A^{(3)}(\phi^{2n+1})$	BChPT Counterparts	Term type: $\mathcal{A}$, $\mathcal{B}$; b_i in EM
11	$[v \cdot \mathrm{D}, [v \cdot \mathrm{D}, \mathrm{S} \cdot u]]$	$\frac{1}{\mathrm{m}^{0^2}} \gamma^5 \left[\left(\mathrm{D}^2 + \mathrm{m}^{0^2} - \left(-(i\slashed{D} - \mathrm{m}^0)^2 \right. \right. \right.$ $\left. \left. + \tfrac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu] \right) \right),$ $\left[\left(\mathrm{D}^2 + \mathrm{m}^{0^2} - \left(-(i\slashed{D} - \mathrm{m}^0)^2 \right. \right. \right.$ $\left. \left. \left. + \tfrac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu] \right) \right), \slashed{u} \right]$	$\mathcal{A}$
12	$(v \cdot \mathrm{D}\, \mathrm{S} \cdot \mathrm{D}\, v \cdot u + \text{h.c.})$	$\frac{1}{\mathrm{m}^{0^2}} \left(\mathrm{D}^2 + \mathrm{m}^{0^2} - \left(-(i\slashed{D} - \mathrm{m}^0)^2 \right. \right.$ $\left. \left. + \tfrac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu] \right) \right) \gamma^5 \slashed{D} [\mathrm{D}^\mu, u_\mu]_+$ $+ \text{h.c.}$	$\mathcal{A}$
13	$[(v \cdot \mathrm{D})^2, \mathrm{S} \cdot u]_+$	$\left[\left(\mathrm{D}^2 + \mathrm{m}^{0^2} - \left(-(i\slashed{D} - \mathrm{m}^0)^2 \right. \right. \right.$ $\left. \left. + \tfrac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu] \right) \right)^2, \gamma^5 \slashed{u} \Big]_+$	$\mathcal{A}$
14	$i\epsilon^{\mu\nu\rho\lambda} v_\mu \mathrm{D}_\nu u_\rho \mathrm{D}_\lambda$	$i\epsilon^{\mu\nu\rho\lambda} \gamma_\mu \mathrm{D}_\nu u_\rho \mathrm{D}_\lambda$	$\mathcal{B}$

i	$A^{(3)}(\phi^{2n})$	BChPT Counterparts	Term type: $\mathcal{A}$, $\mathcal{B}$; b_i in EM
15	$i[v \cdot \mathrm{D}, u^2]_+$	$\frac{1}{\mathrm{m}^0}\left[\left(\mathrm{D}^2 + \mathrm{m}^{0\,2} - \left(-(i\not{D} - \mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right), u^2\right]_+$	$\mathcal{A}$
16	$i[v \cdot \mathrm{D}, (v \cdot u)^2]_+$	$\frac{1}{\mathrm{m}^{0\,2}}\left[\left(\mathrm{D}^2 + \mathrm{m}^{0\,2} - \left(-(i\not{D} - \mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right), [\mathrm{D}^\mu, u_\mu]_+^2\right]_+$	$\mathcal{A}$
17	$i[v \cdot u, [v \cdot \mathrm{D}, v \cdot u]]$	$\frac{1}{\mathrm{m}^{0\,2}}\left[[\mathrm{D}_\mu, u^\mu]_+, \left[\left(\mathrm{D}^2 + \mathrm{m}^{0\,2} - \left(-(i\not{D} - \mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right), [\mathrm{D}_\nu, u^\nu]_+\right]\right]$	b_3
18	$\frac{1}{\mathrm{m}^0}[u^\mu, [v \cdot \mathrm{D}, u_\mu]]$	$\frac{1}{\mathrm{m}^0}\left[u^\mu, \left[\left([\mathrm{D}^2 + \mathrm{m}^{0\,2} - \left(-(i\not{D} - \mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right), u_\mu\right]\right]$	b_1
19	$\epsilon^{\mu\nu\rho\lambda}v_\rho \mathrm{S}_\lambda[[u_\mu, u_\nu], v \cdot \mathrm{D}]_+$	$\frac{i}{\mathrm{m}^0}\sigma^{\mu\nu}\left[[u_\mu, u_\nu], \left(\mathrm{D}^2 + \mathrm{m}^{0\,2} - \left(-(i\not{D} - \mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right)\right]_+$	$\mathcal{A}$

i	$A^{(3)}(\phi^{2n})$	BChPT Counterparts	Term type: $\mathcal{A}$, $\mathcal{B}$; b_i in EM
20	$\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda[u_\mu,[v\cdot D,u_\nu]]_+$	$\sigma^{\mu\nu}\left[u^\mu,\left[\left[D^2+m^{0^2}-\left(-(i\slashed{D}-m^0)^2+\frac{i}{8}\sigma^{\mu\nu}[u_\mu,u_\nu]\right)\right],u_\nu\right]\right]_+$	b_{15}
21	$(iu^\mu v\cdot uD_\mu+\text{h.c.})$	$(iu^\mu\slashed{u}D_\mu+\frac{1}{m^0}(u\cdot D)^2)+\text{h.c.}$	b_4
22	$i[u^2,v\cdot D]_+$	$\frac{1}{m^0}\left[u^2,\left(D^2+m^{0^2}-\left(-(i\slashed{D}-m^0)^2+\frac{i}{8}\sigma^{\mu\nu}[u_\mu,u_\nu]\right)\right)\right]_+$	$\mathcal{A}$
23	$i[u_\mu,[v\cdot u,D^\mu]]$	$i[u^\mu,[\slashed{u},D_\mu]]$	b_2
24	$\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda[D_\mu,[u_\nu,v\cdot u]]_+$	$\frac{1}{m^{0^2}}\epsilon^{\mu\nu\rho\lambda}\gamma^5\gamma_\lambda[D_\rho,[u_\nu,[D_\kappa,u^\kappa]_+]]_+,]_+$	$\mathcal{B}$
25	$\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda[u_\mu,[D_\nu,v\cdot u]]_+$	$\frac{1}{m^{0^2}}\epsilon^{\mu\nu\rho\lambda}\gamma^5\gamma_\lambda[D_\rho,[u_\mu,[D_\nu,[D_\alpha,u^\alpha]_+]]]_+]_+$	b_{16}
26	$\epsilon^{\mu\nu\rho\lambda}v_\rho S_\lambda[[D_\mu,u_\nu],v\cdot u]_+$	$\frac{1}{m^{0^2}}\epsilon^{\mu\nu\rho\lambda}\gamma^5\gamma_\lambda[D_\rho,[[D_\mu,u_\nu],[D_\alpha,u^\alpha]_+]_+]_+$	$\mathcal{B}$
27	$\epsilon^{\mu\nu\rho\lambda}S_\rho[D_\mu,[u_\nu,u_\lambda]]_+$	$\epsilon^{\mu\nu\rho\lambda}\gamma^5\gamma_\rho[D_\mu,[u_\nu,u_\lambda]]_+$ $+2im^0\sigma^{\mu\nu}[u_\mu,u_\nu]$	$\mathcal{B}$
28	$i(v\cdot D)^3$	$\frac{1}{m^{0^3}}\left(D^2+m^{0^2}-\left(-(i\slashed{D}-m^0)^2+\frac{i}{8}\sigma^{\mu\nu}[u_\mu,u_\nu]\right)\right)^3$	$\mathcal{A}$

i	$A^{(3)}(\phi^{2n})$	BChPT Counterparts	Term type: $\mathcal{A}$, $\mathcal{B}$; b_i in EM
29	$i[\mathrm{D}^2, v \cdot \mathrm{D}]_+$	$\frac{1}{\mathrm{m}^0}\left[\left(\mathrm{D}^2 + \mathrm{m}^{02} - \left(-(i\displaystyle{\not}D - \mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right), \left(-(i\displaystyle{\not}D - \mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right)\right]_+$	$\mathcal{A}$
30	$[\chi_-, v \cdot u]$	$\frac{i}{\mathrm{m}^0}[\chi_-, [\mathrm{D}_\mu, u^\mu]_+]$	b_6
31	$i[v \cdot \mathrm{D}, \chi_+]_+$	$\frac{1}{\mathrm{m}^0}\left[\left(-(i\displaystyle{\not}D - \mathrm{m}^0)^2 + \frac{i}{8}\sigma^{\mu\nu}[u_\mu, u_\nu]\right), \chi_+\right]_+$	$\mathcal{A}$